MID-SEMESTER EXAMINATION
ANALYSIS III, B. MATH II YEAR
I SEMESTER, 2010-2011

Max. you can score: 80 Time limit: 3 hrs.

1. Prove that the following statements are equivalent for any continuous function $g : [a, b] \to \mathbb{R}$:

a) for every continuous map $f : [a, b] \to \mathbb{R}$ and every $\epsilon > 0$ there exists a polynomial p such that $|f(x) - p(g(x))| < \epsilon$ for all $x \in [a, b]$.

b) g is one-to-one on $[a, b]$ [20]

2. Let $f(y) = \int_0^\infty (e^{-xy})(\frac{\sin x}{x})dx$ if $y > 0$ and $f(0) = \lim_{N\to\infty} \int_0^N \frac{\sin x}{x} dx$. [Assume that the limit exists]. Prove that f is continuous on $[0, \infty)$ and that $f(y) \to 0$ as $y \to \infty$. [20]

3. Prove or disprove the following:

the series $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ...$ converges uniformly on $\mathbb{R}$. [5]

4.

a) Find the curvature at any point for the curve $\phi(t) = (t, \log(\cos t)), -\pi/4 \leq t \leq \pi/4$. [10]

b) Show that a smooth curve ϕ in $\mathbb{R}^2$ is (a segment of) a straight line if and only if the tangent lines to the curve are all parallel to each other. [10]

5. Let $\phi(t) = (3 \cos t, 3 \sin t, 8t), 0 \leq t \leq 1$. Prove that the torsion $\tau(t)$ is a constant and find the value of this constant. [15]